

# A graphical language for beam splitters

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We introduce a graphical language based on a traced PROP to describe a family of systems in which a photon (or a beam) is sent in superposition of several positions and carries some additional quantum data represented by a qudit. Our systems are composed of polarizing beam splitters and black box unitary transformations on the data carried by the photon. This language can be used to represent protocols with indefinite causal order, including the so-called quantum switch.

Our language is equipped with a categorical denotational semantics and a simple complete axiomatization, the proof of completeness being based on a normal form. We also introduce a colored variant equipped with an analogous semantics and another complete axiomatization, which allows us to remove the useless wires, and has a simpler normal form and proof of completeness. Finally, we prove that extending our language with standard non-polarizing beam splitters allow us to perform any unitary transformation on the whole state of the photon.



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