Quantitative estimation of the evolution of entanglement in the Grover algorithm

Henri de Boutray, Alain Giorgetti, Frédérick Holweck, Pierre-Alain Masson and Hamza Jaffali

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Quantum entanglement has been identified as a key ingredient in the speed-up of quantum algorithms [JL03], such as Grover algorithm [Gro96]. However, the precise contribution of entanglement in Grover algorithm is not yet well understood. Previous works tackled entanglement in the Grover algorithm from two perspectives: quantitatively, with the Geometric Measure of Entanglement [WG03, RBM13] or Mermin polynomials [BOF+16], and qualitatively, by observing the different entanglement SLOCC classes traversed by its execution [JH19].

We study the evolution of entanglement in Grover algorithm with Mermin polynomials [Mer90, ACG+16, AL16] because, as a generalization of CHSH inequalities [CHSH69], they have two nice features: their evaluation of quantum states can be compared to a classical bound, and a physical implementation of their evaluation is theoretically feasible. With this choice our work is close to the work of Batle et al. [BOF+16] but it adds some novelties in the techniques of evaluation, making them more efficient. We also manage to break the classical limit. With this study, we provide some code that simulates the execution of the Grover algorithm and evaluates the entanglement at each iteration. A particular attention has been given make this code it as reliable and reusable as possible.

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