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Controllability & Optimal Control of Spins Coupled to a Dissipative Cavity

GdR IM
Project I-QUINS

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Contents

- Context
- Model systems
- Controllability
- Applications
- Conclusion

→ two papers in preparation,
Available soon on ArXiv & Research gate

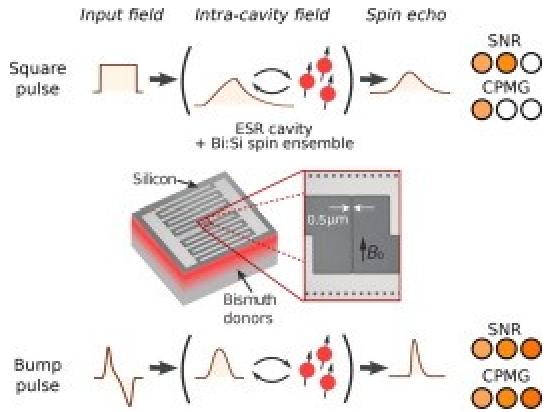
Context

Recent collaboration with the Quantronics Group (CEA-Saclay), in particular with S. Probst and P. Bertet.

→ Experiments involving an ensemble of spins coupled to a microwave resonator

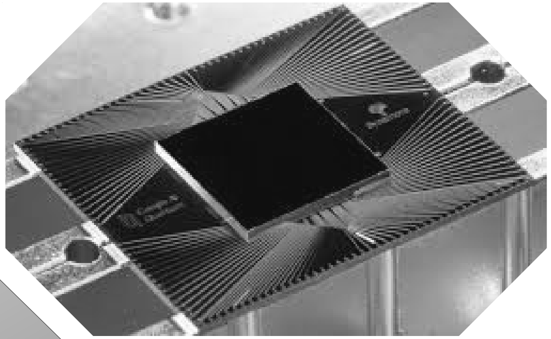
S. Probst & al. *Shaped pulses for transient compensation in quantum-limited electron spin resonance spectroscopy*, JMR 303, 42-47 (2019)

Q. Ansel & al. *Optimal control of an inhomogeneous spin ensemble coupled to a cavity*, Phys. Rev. A 98, 023425 (2018)



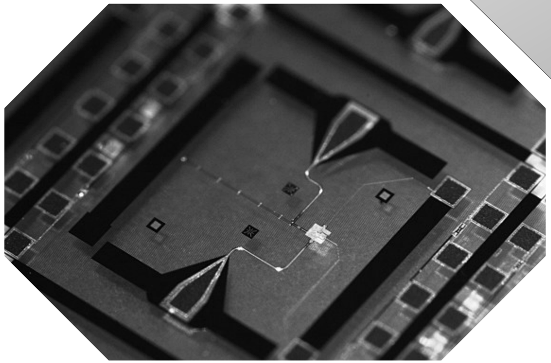
Context

→ Quantum measurements, quantum sensing.
→ Generation of entangled states.



Quantum Technologies

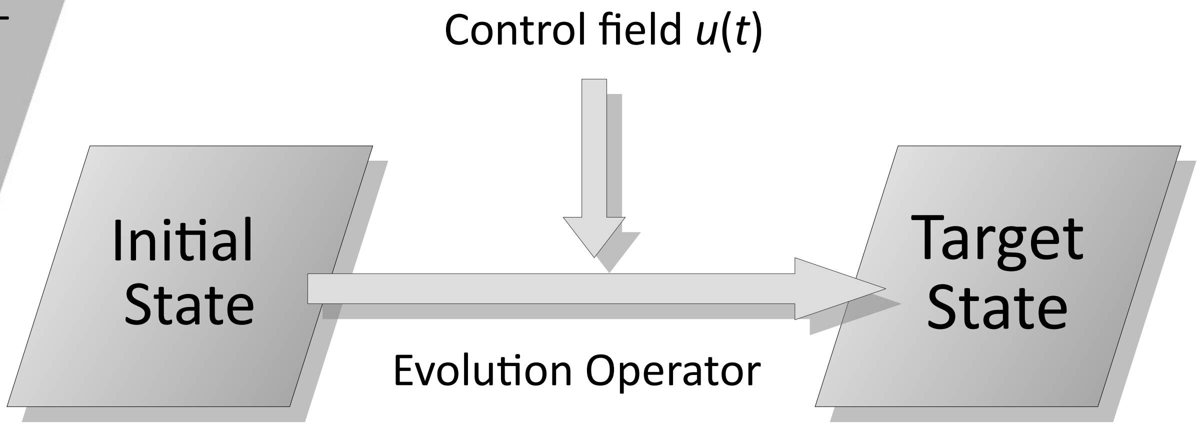
Fundamental Questions



→ to what extent can we control an open quantum system?
→ to what extent can we generate entanglement in a system with dissipation?

Quantum Control

- What is a quantum control-optimal problem ?



- Goal: find $u(t)$ that minimizes some constraint(s) (e.g. control time)

  → analytic expression (if lucky)

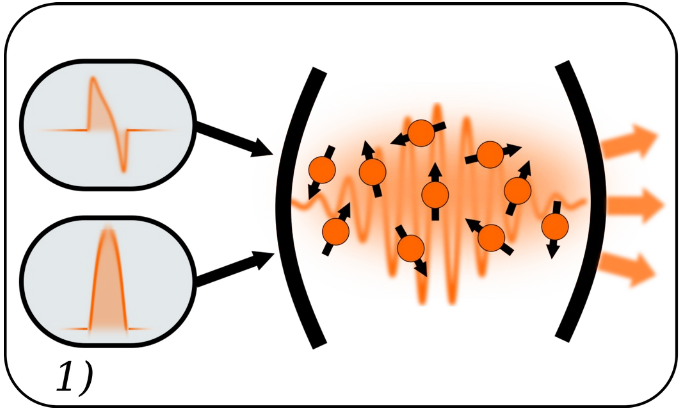
  → numerical optimization (otherwise)

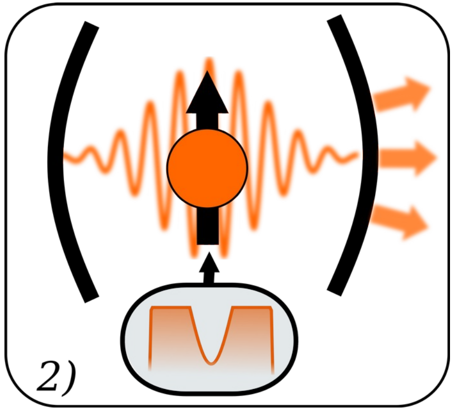
Two model systems

- Ensemble of spins ½
- Damped cavity mode
- Jaynes-Cummings interaction
- Control with coherent and squeezing controls on the cavity mode

- Single spin ½ and *max* 1 quantum excitation.
- Damped cavity mode
- Jaynes-Cummings interaction
- Control of the spin energy transition.

→ Phys. Rev. X 7,041011 (2017)
→ A. Bienfait, PhD thesis
→ Appl. Phys. Lett. 111, 202604 (2017).





→ Phys. Rev. A 61, 025802 (2000)

→ Phys. B 27, 1345053 (2013).

→ Sci. Rep. 8, 1 (2018)

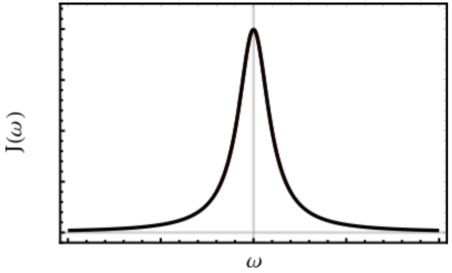
Two model systems

$$\frac{d\hat{\rho}}{dt} = -\frac{i}{\hbar}[\hat{H}(t),\hat{\rho}] + \kappa\left[\hat{a}\hat{\rho}\hat{a}^\dagger - \frac{1}{2}\left(\hat{a}^\dagger\hat{a}\hat{\rho} + \hat{\rho}\hat{a}^\dagger\hat{a}\right)\right]$$

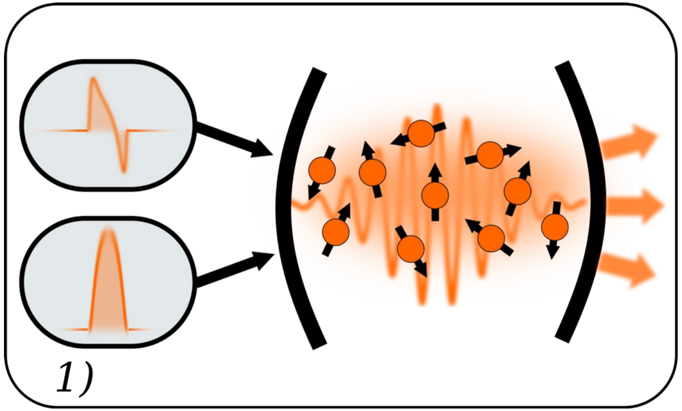
$$\hat{H}(t) = \underbrace{i\hbar\left[\beta(t)\hat{a}^\dagger - \beta(t)^*\hat{a}\right] + \alpha(t)\frac{i\hbar}{2}\left[(\hat{a}^\dagger)^2 - (\hat{a})^2\right]}_{\hat{H}_C} \\ \underbrace{+ \sum_{n=1}^{N_s}\frac{\hbar\Delta_n}{2}\hat{\sigma}_z^{(n)} + \hbar g\sum_{n=1}^{N_s}\left[\hat{a}^\dagger\hat{\sigma}_-^{(n)} + \hat{a}\hat{\sigma}_+^{(n)}\right]}_{\hat{H}_0}$$

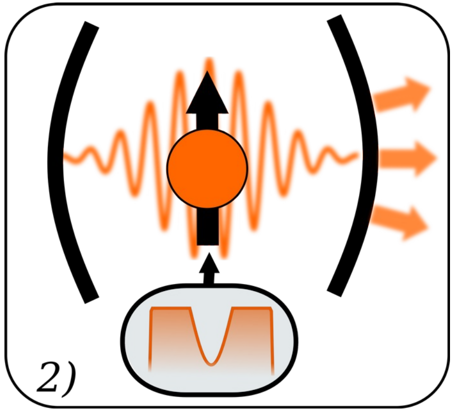
$$\hat{H}(t) = \hbar\left(\omega_0(t)\hat{\sigma}_+\hat{\sigma}_- + \sum_l\left[\omega_l\hat{a}_l^\dagger\hat{a}_l + g_l\hat{\sigma}_+\hat{a}_l + g_l^*\hat{\sigma}_-\hat{a}_l^\dagger\right]\right)$$

$+$



Lorentzian distribution of mode → effective cavity mode

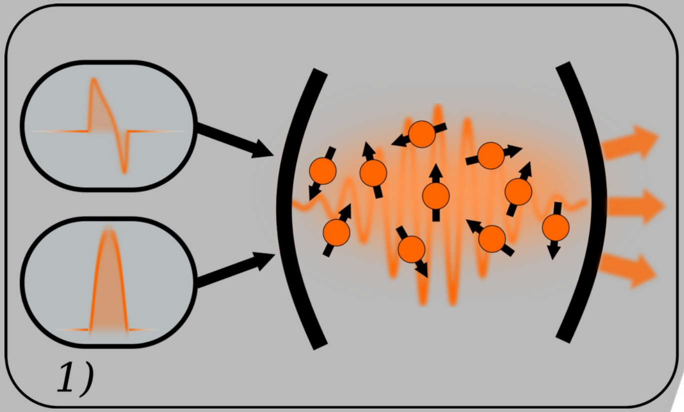


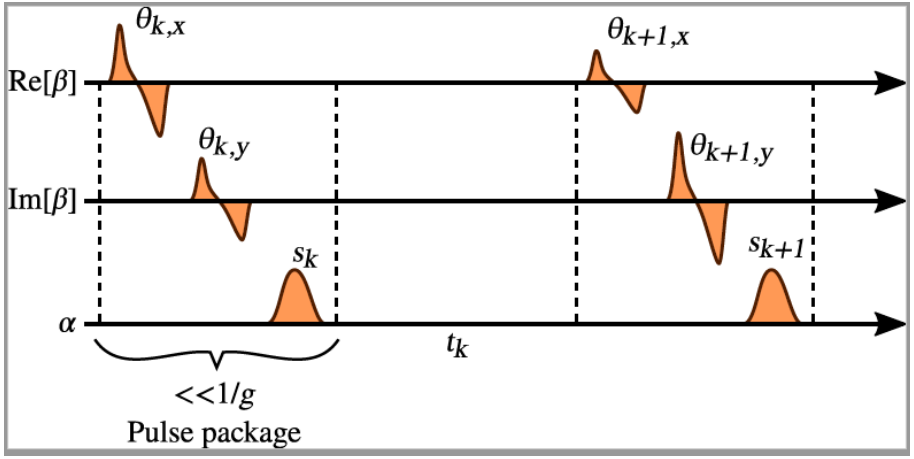


Model 1

→ Use of pulse sequences

Each pulse is parameterized by its amplitude and its position in the sequence → parameters to optimize numerically.

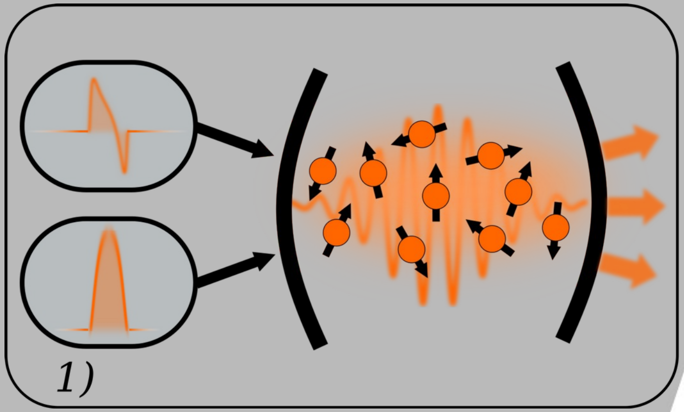




Use at most 7 pulse packages → max 28 parameters to determine

→Further details: Phys. Rev. A 98, 023425(2018), Q. Ansel, PhD thesis

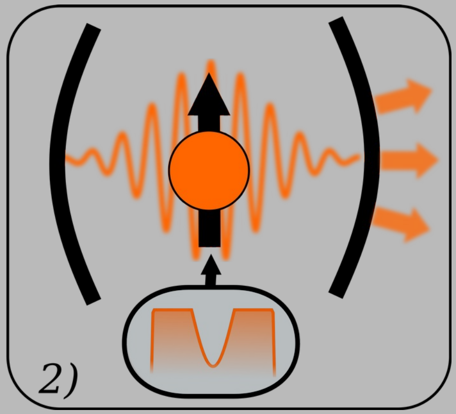
Model 1



- Collective controls on the spin ensemble.
- Spins cannot be controlled individually
- → Make the control task very difficult...

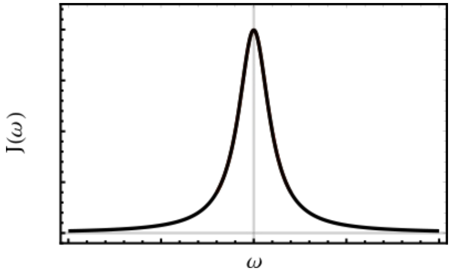
Model 2

Change of variables allows us to simplify drastically the system complexity.



$$\hat{H}(t) = \hbar\left(\omega_0(t)\hat{\sigma}_+\hat{\sigma}_- + \sum_l \left[\omega_l \hat{a}_l^\dagger \hat{a}_l + g_l \hat{\sigma}_+ \hat{a}_l + g_l^* \hat{\sigma}_- \hat{a}_l^\dagger\right]\right)$$

+

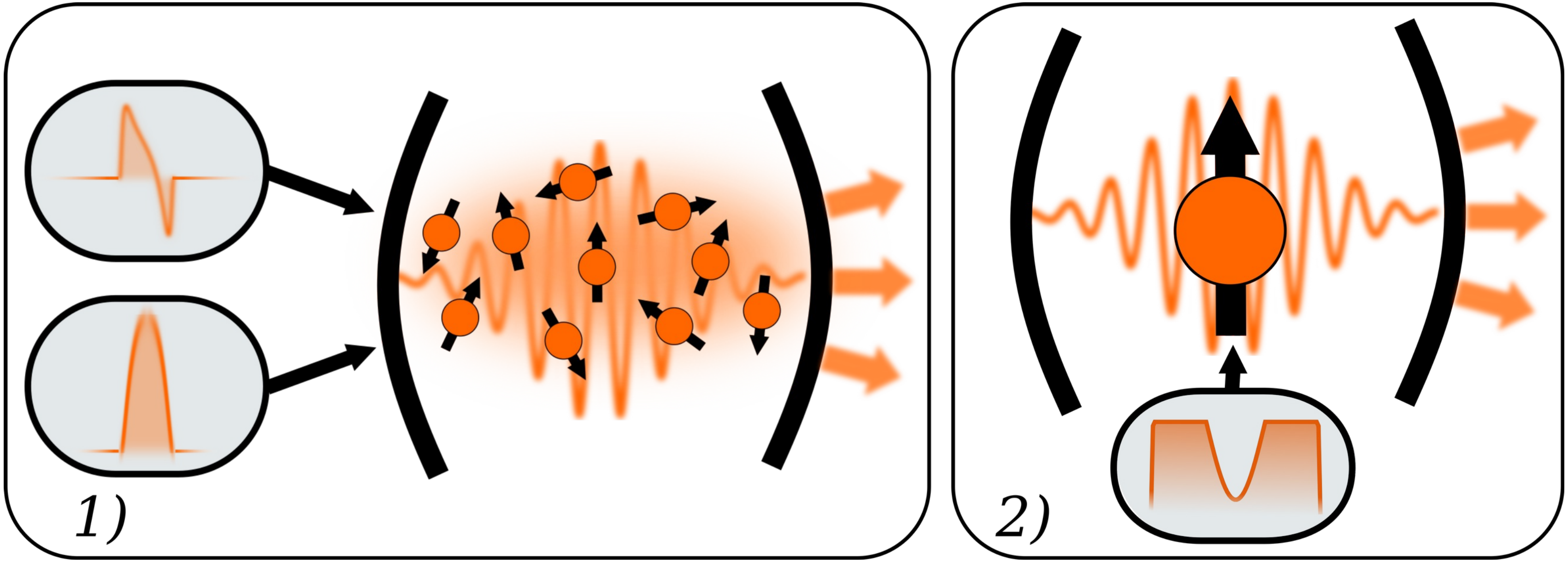


Lorentzian distribution of mode → effective cavity mode

Max 1 quantum excitation.

$$\frac{d}{dt}\begin{pmatrix} c_1 \\ y \end{pmatrix} = \begin{pmatrix} -i\omega(t) & -p \\ p & -q \end{pmatrix}\begin{pmatrix} c_1 \\ y \end{pmatrix}$$

Two model systems



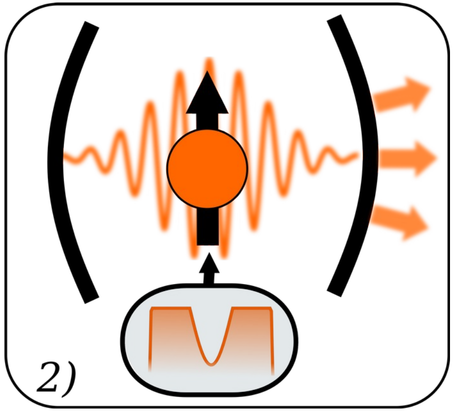
Controllability ?

- <u>Definition:</u> Complete state controllability describes the ability of an external input (the control field(s)) to move the state of a system **from any** *initial state* **to any** other *final state*.

- ***Are the systems controllable ?***

Model 2

- The system is described by a liner time-dependent ODE→ controllability can be studied using Lie algebra methods.
- Complete controllability if the group GL(2,C) can be generated
- → simple calculation shows that the reachable set is:



$$\mathcal{R}(t) = \left\{ \begin{pmatrix} e^{-iW(t')} & 0 \\ 0 & e^{-qt'} \end{pmatrix} h(W(t')), \right.$$

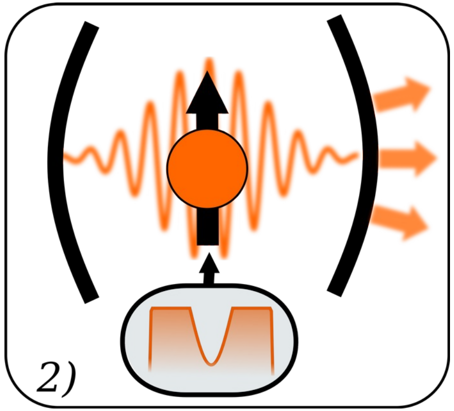
$$\text{such that } h \in \mathsf{H} \subseteq SL(2\mathbb{C}), \text{ and } t' \leq t\}$$

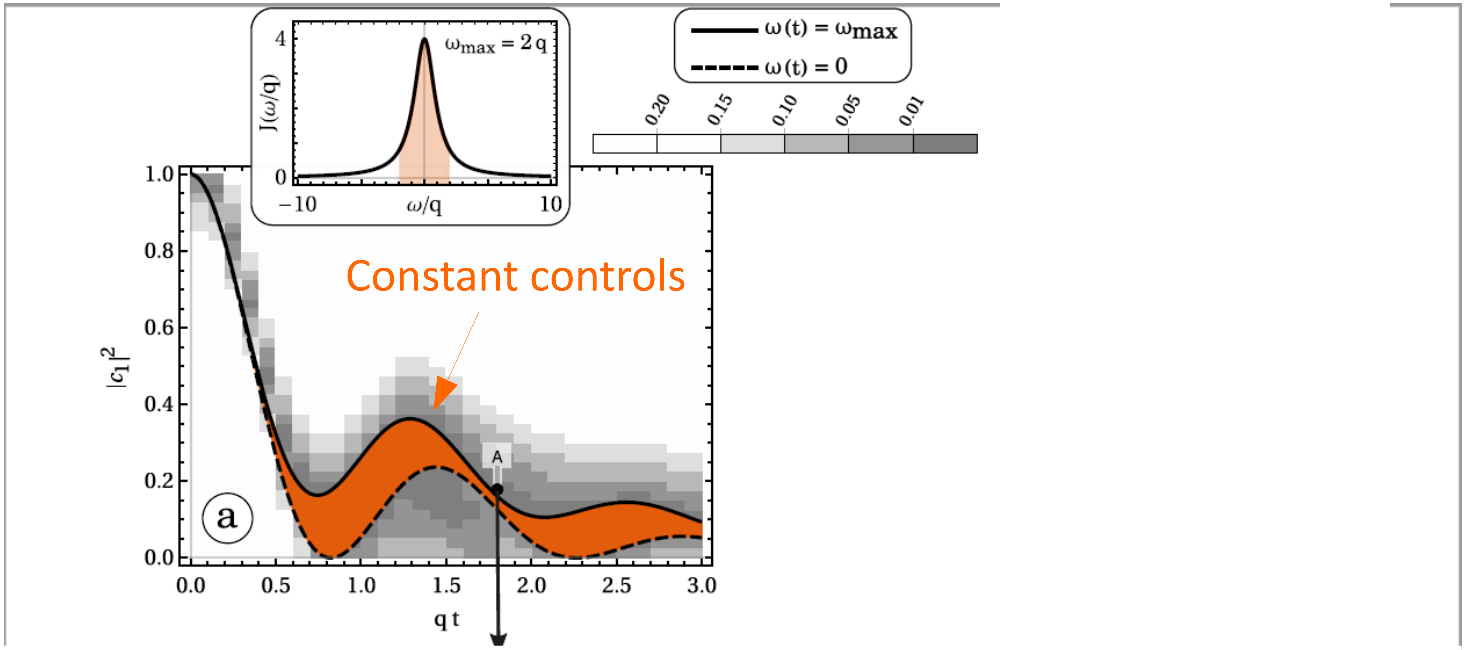
$$\frac{d}{dt}\begin{pmatrix} c_1 \\ y \end{pmatrix} = \begin{pmatrix} -i\omega(t) & -p \\ p & -q \end{pmatrix}\begin{pmatrix} c_1 \\ y \end{pmatrix}$$

Not controllable !

Model 2

Okay, model 2 is not controllable, but what are the spin states that can be reached?





Numerical search of the reachable points using control field optimization. Dark gray → the target state can be reached with a high probability !

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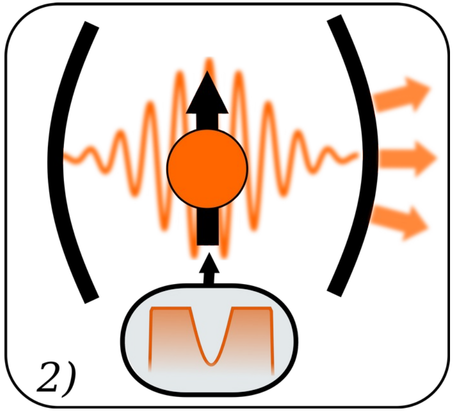

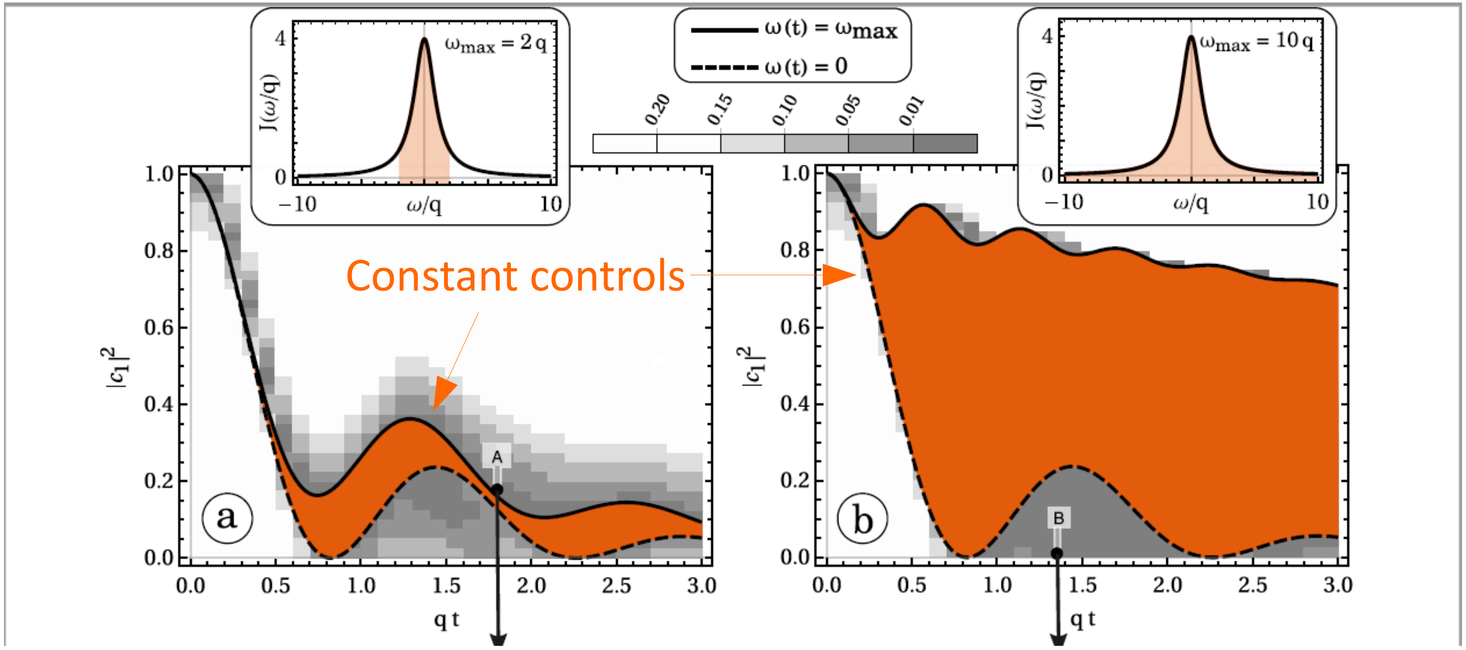

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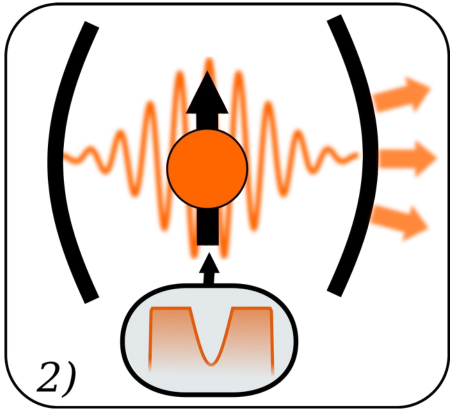

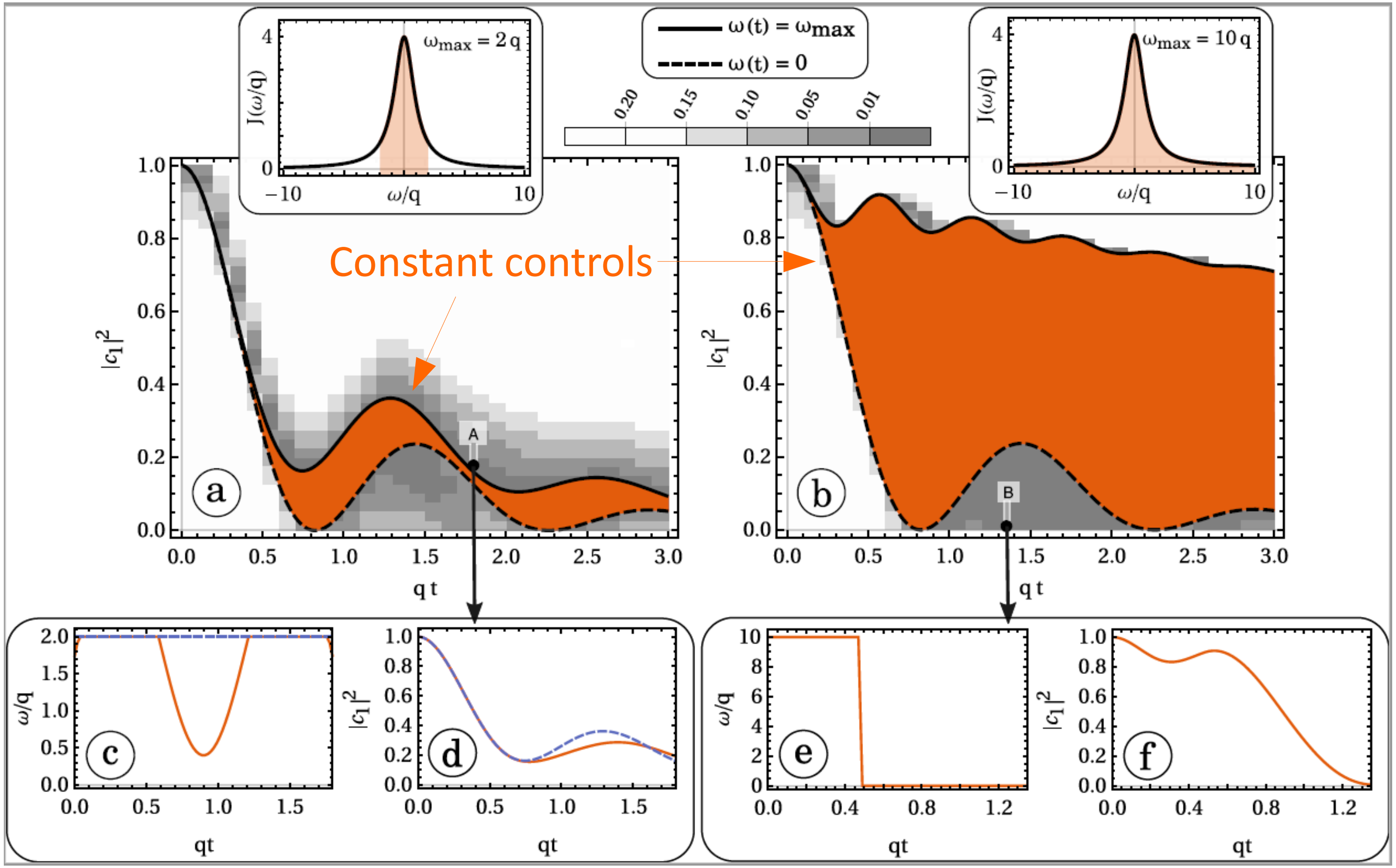

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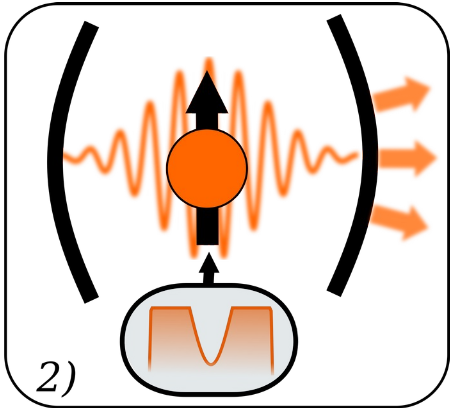
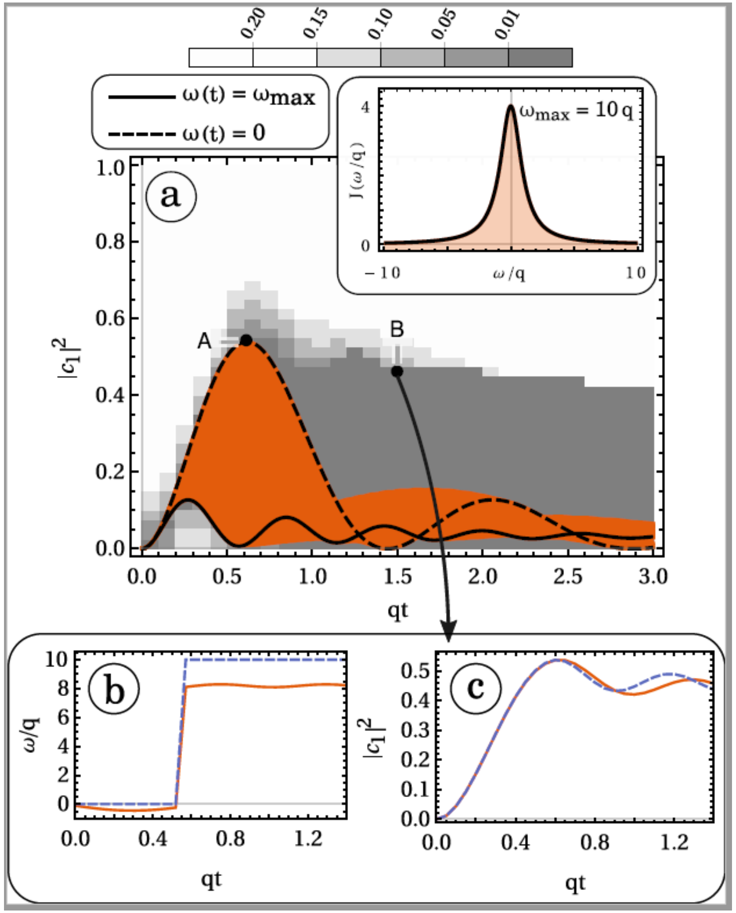
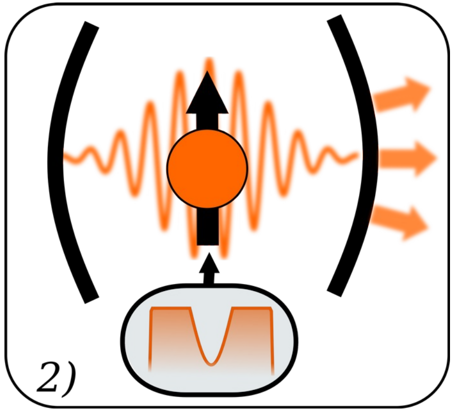




Same as before, but with another initial state.

Model 2

- Most of the points can be reached with simple controls (analytic formulas).
- Inhibition of the spin decay: modulated controls gives a modest improvement than the detuning effect.
- Efficient controls to drive the spin on the ground state.



Model 1

→ *no relaxation* : system is controllable.

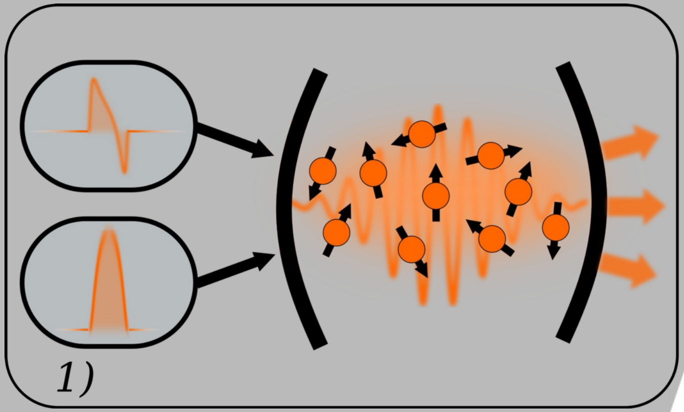
→ Set of possible transformations: SU(N)

→ Squeezing field: faster generation of the dynamical Lie algebra.

Commutator Order	0	1	2	3	4	5
$(\hat{H}_0, \hat{V}_x, \hat{V}_y)$	3	6	12	44	288	323
$(\hat{H}_0, \boxed{\hat{V}_x, \hat{V}_y}, \boxed{\hat{V}_s})$	4	8	21	138	323	323

Coherent control terms

Squeezing control terms

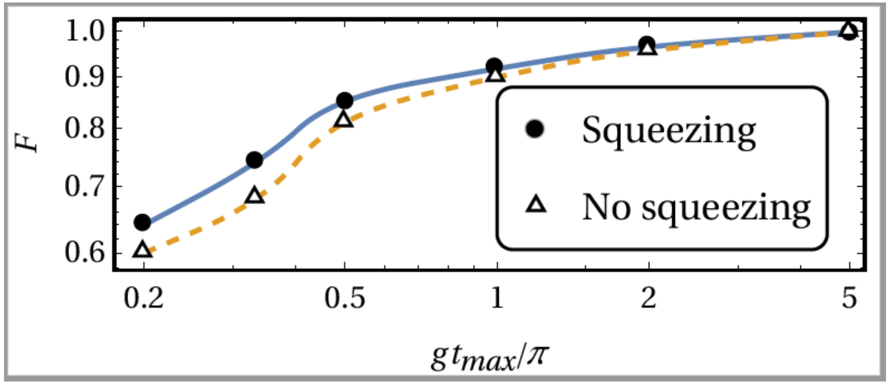


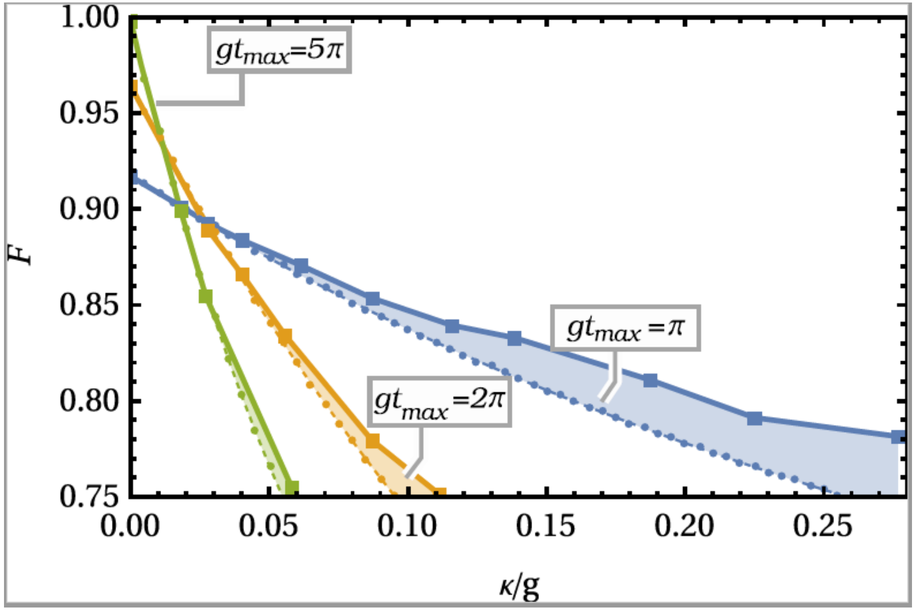
Dimensions of the Lie algebra spanned by recursive commutators of the Hamiltonians. The commutator order corresponds to the maximum number of commutators taken into account. Calculations performed on a truncated Hilbert space.

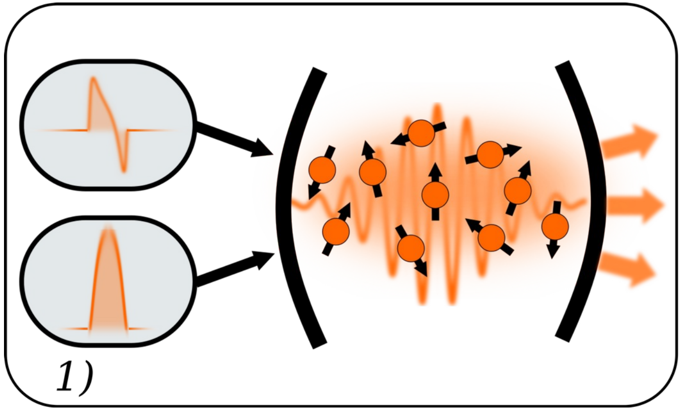
DimH = 18, → dim(su(18)) = N^2-1 = 323.

Applications

Generation of a symmetric state (2 spins).







$$\begin{matrix} |\downarrow,\downarrow\rangle \\ |0\rangle \end{matrix} \longrightarrow \begin{matrix} (|\uparrow,\downarrow\rangle + |\downarrow,\uparrow\rangle)/\sqrt{2} \\ |0\rangle \end{matrix}$$

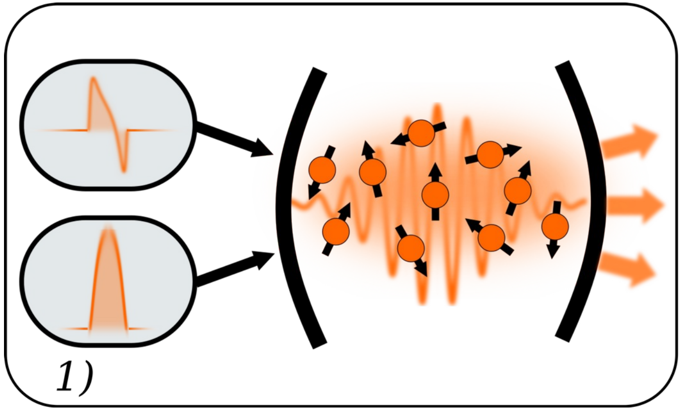
Spins

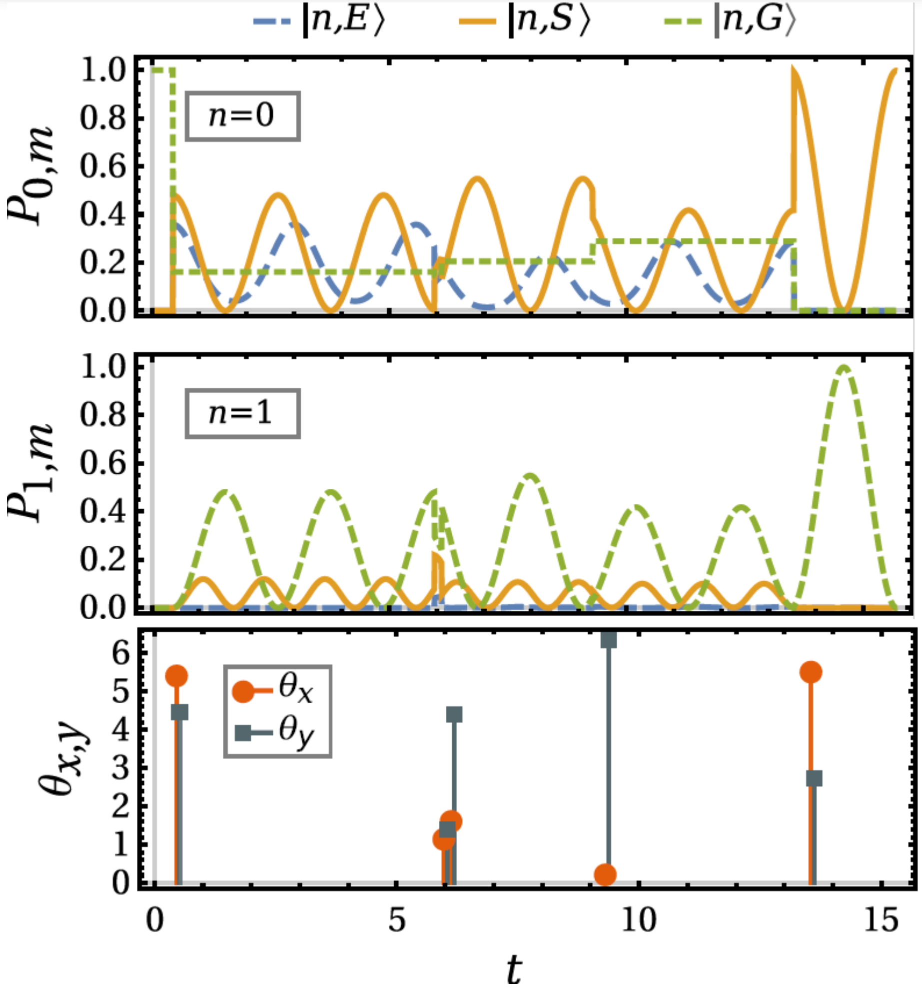
Cavity

Applications

Optimal solution without cavity damping:

$$\begin{Bmatrix} |\downarrow,\downarrow\rangle \\ |0\rangle \end{Bmatrix} \rightarrow \begin{Bmatrix} (|\uparrow,\downarrow\rangle + |\downarrow,\uparrow\rangle)/\sqrt{2} \\ |0\rangle \end{Bmatrix}$$





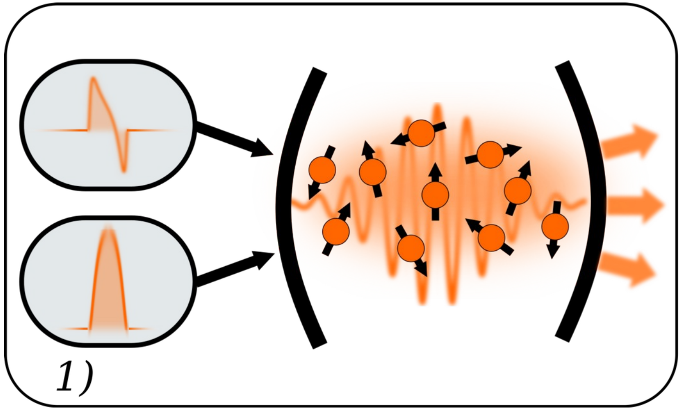
Applications

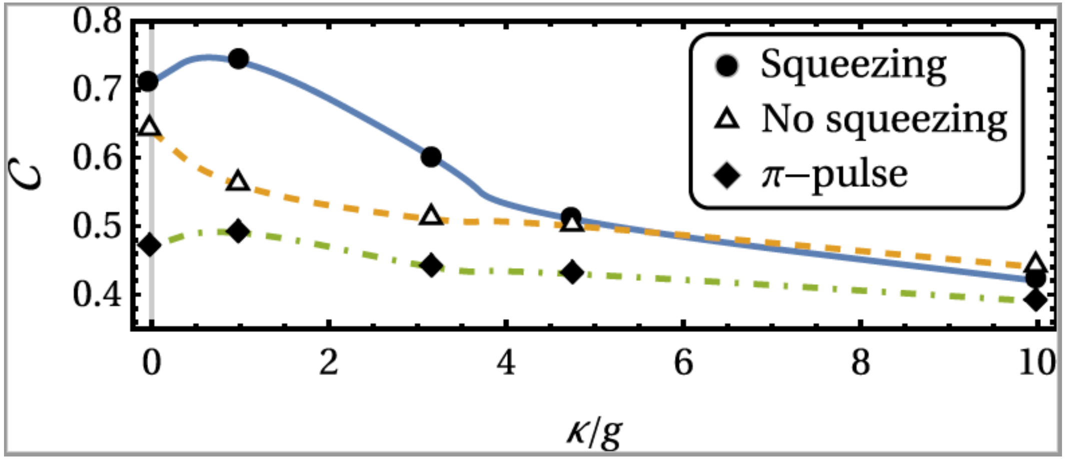
Generation of entangled states with a measure of non-classicality.

$$\mathcal{C}(t_f) = \frac{8}{N_s^2}\sum_{m<n}\langle\hat{\sigma}_+^{(n)}\hat{\sigma}_-^{(m)}\rangle_c(t_f)$$
$$= \frac{8}{N_s^2}\sum_{m<n}\langle\hat{\sigma}_+^{(n)}\hat{\sigma}_-^{(m)}\rangle(t_f) - \langle\hat{\sigma}_+^{(n)}\rangle(t_f)\langle\hat{\sigma}_-^{(m)}\rangle(t_f)$$

- Ensemble of 4 spins.
- Short control duration (t_{max}=π/2).
- Detuning distribution:

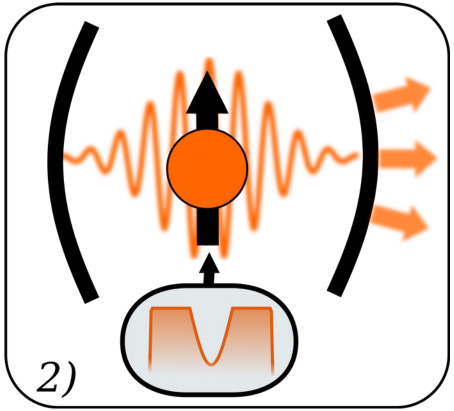
$\Delta_n/g \in \{-1,-0.5,0.5,1\}$

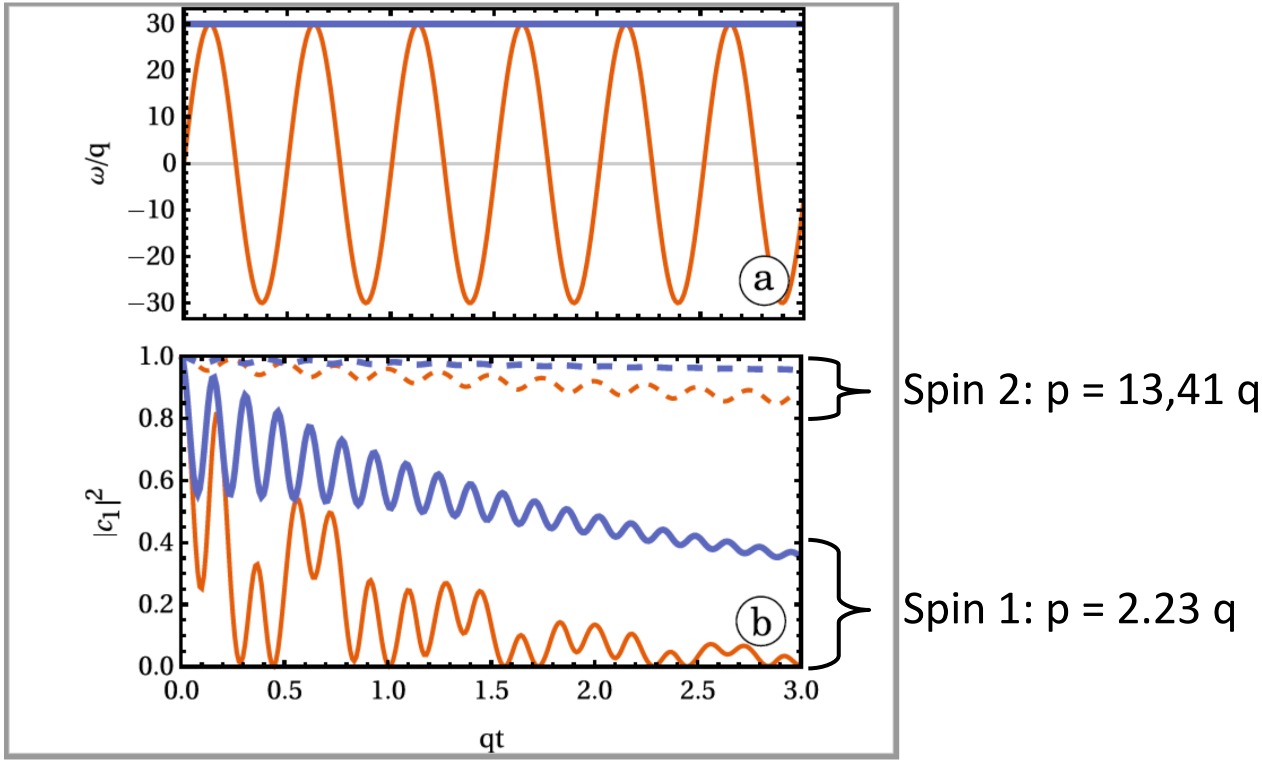




Applications

Selectivity of distinct spins with constant or sinusoidal controls.





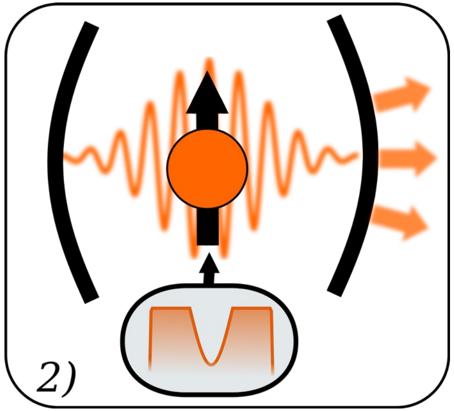
Applications

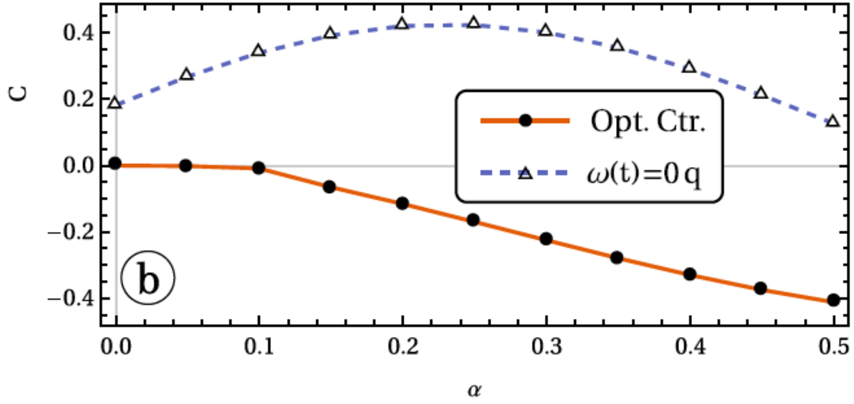
The selectivity process can be optimized.

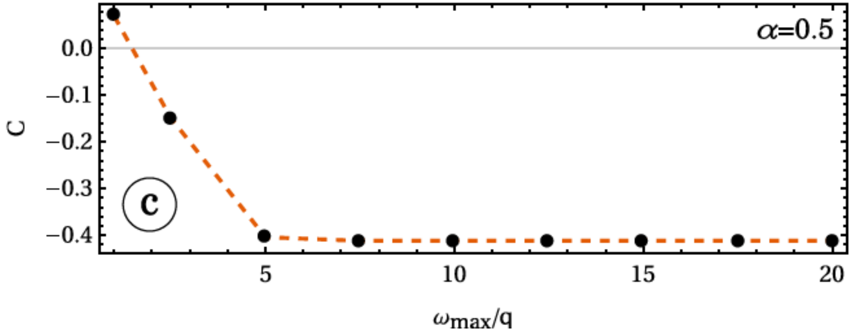
Cost function used in the calculations:

$$C = \lambda |c_1^{(2)}(t_f)|^2 - |c_1^{(1)}(t_f)|^2$$

$$p^{(2)} = p^{(1)}(1+\alpha),\ p^{(1)} = \sqrt{5}q)$$







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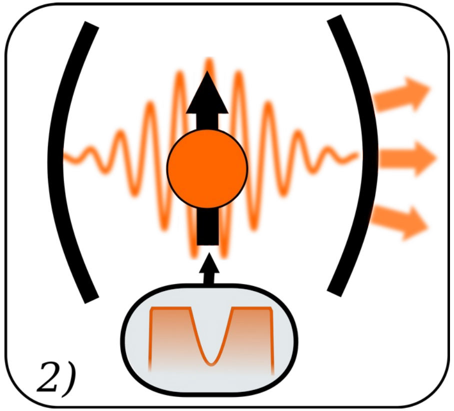

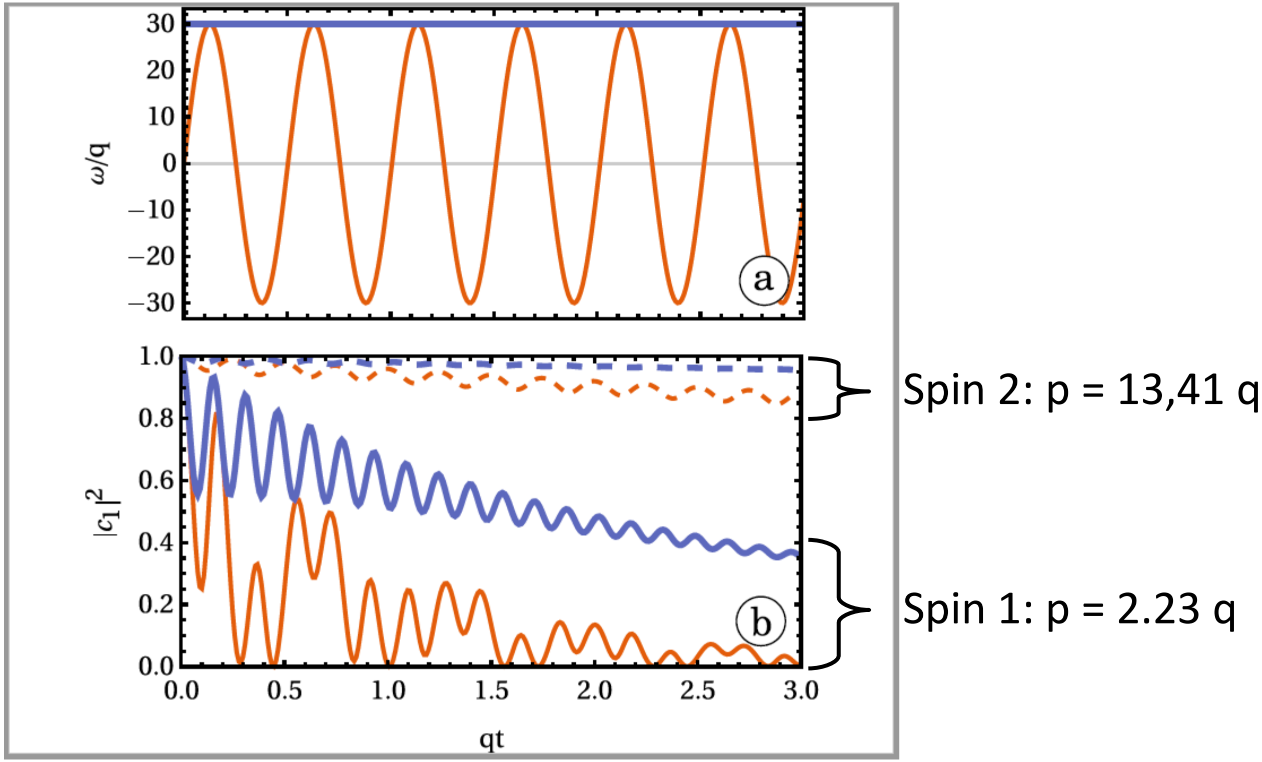

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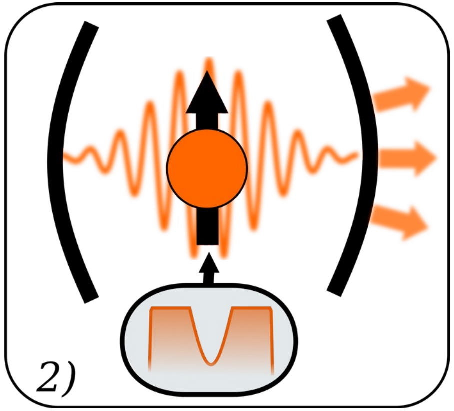
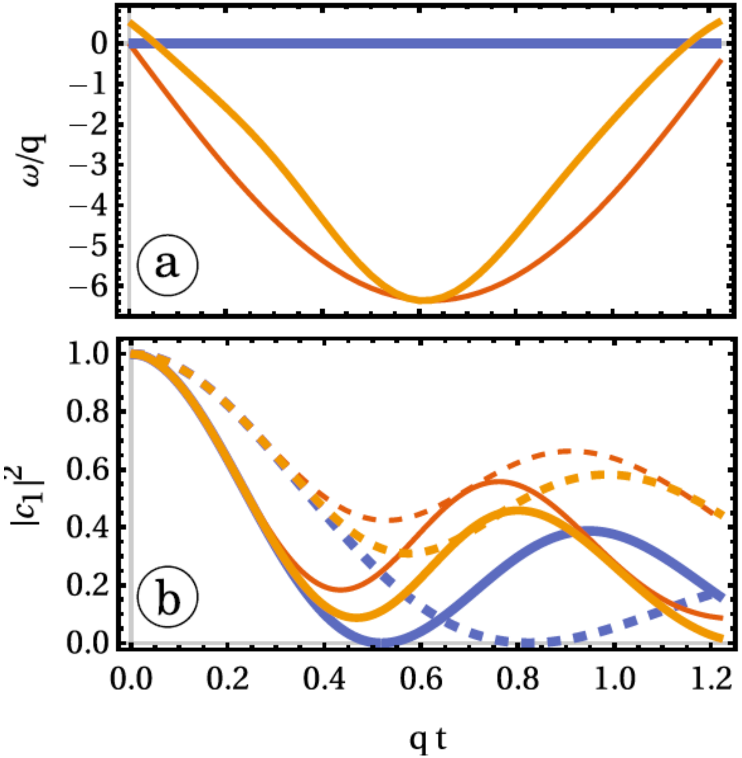




Conclusion

Explore the physical limits of these control problems.

Controllability of Spin systems coupled to a dissipative environement

- ✗ No full controllability
- ✓ Simple control mechanisms (model 2)
- ✓ Squeezing enhance the generation of the dynamic Lie algebra (model 1)

Generation of Entangled state

- ✗ Strongly limited by the relaxation
- ✓ Squeezing provides Better results with a measure of non classicality

Selectivity

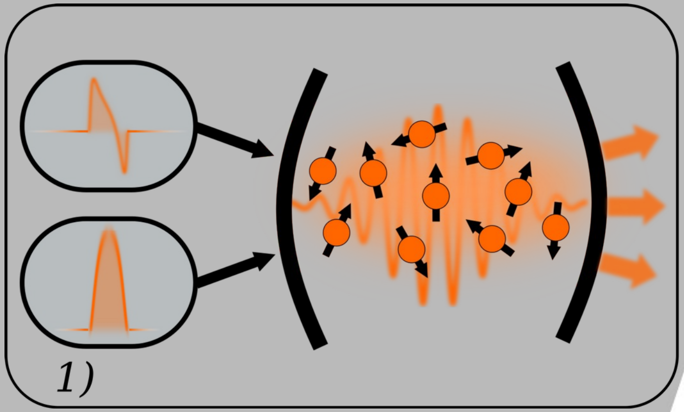
- ✓ Optimal control Allows to reach the Physical limit of parameter selectivity

Model 1

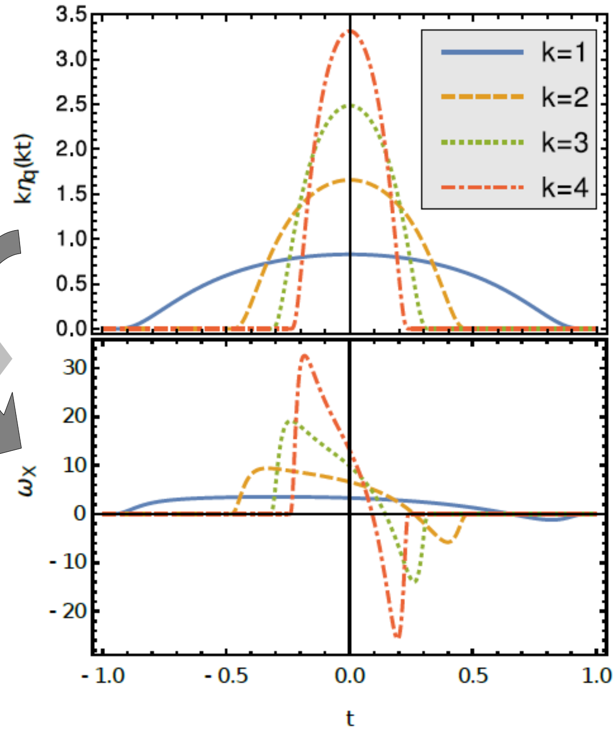
Use of bump pulse (coherent control)

→ Short pulse approximation

→ simplify the numerical calculations.



Transformation,
To take into account
The cavity dynamics



→ Phys. Rev. A 98, 023425(2018)
→ Q. Ansel, PhD thesis