Titouan Carette, LORIA, Université de Lorraine
Dominic Horsman, Université de grenoble
Simon Perdrix, LORIA, CNRS

presents

SZX-calculus, Scalable Graphical Quantum Reasoning

A GTIQ2019 talk

Picturing processes

Name	Symbol	Diagram	Hilbert space

Picturing processes

Name	Symbol	Diagram	Hilbert space
Transformation	f		

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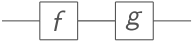
Composition	$g \circ f$		matrix product

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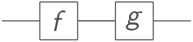
Composition	$g \circ f$		matrix product
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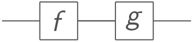
Composition	$g \circ f$		matrix product
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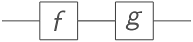
Composition	$g \circ f$		matrix product
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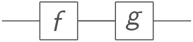
Composition	$g \circ f$		matrix product
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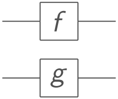
Tensor	$f \otimes g$		Kronecker product

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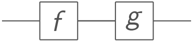
Composition	$g \circ f$		matrix product
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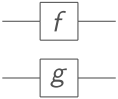
Tensor	$f \otimes g$		Kronecker product
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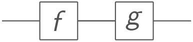
Composition	$g \circ f$		matrix product
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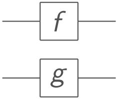
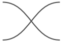
Tensor	$f \otimes g$		Kronecker product
Swap	σ		Basis permutation

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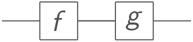
Composition	$g \circ f$		matrix product
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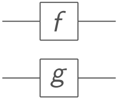
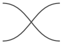
Tensor	$f \otimes g$		Kronecker product
Swap	σ		Basis permutation
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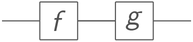
Composition	$g \circ f$		matrix product
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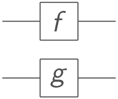
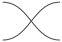
Tensor	$f \otimes g$		Kronecker product
Swap	σ		Basis permutation
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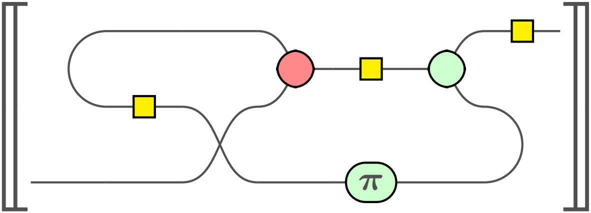
But we only need a small part of it!

ZX-calculus



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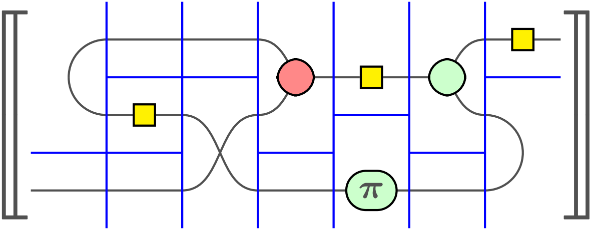

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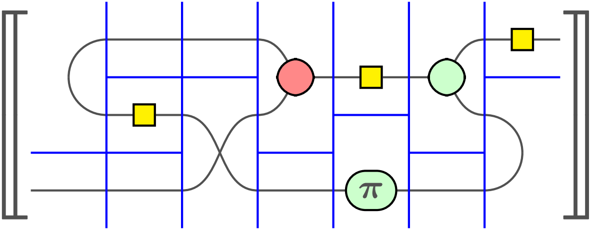
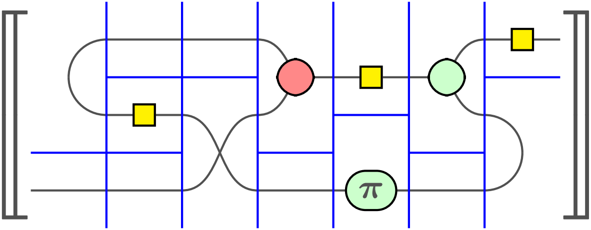


$\left[H \otimes \begin{pmatrix} 1 & 0 & 0 & 1 \end{pmatrix}\right]$

ZX-calculus



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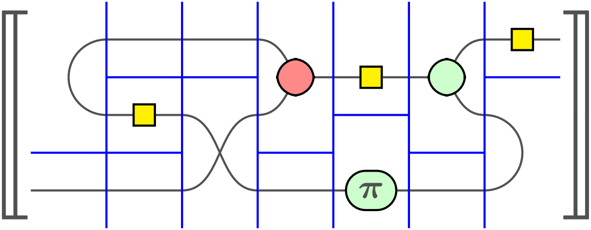

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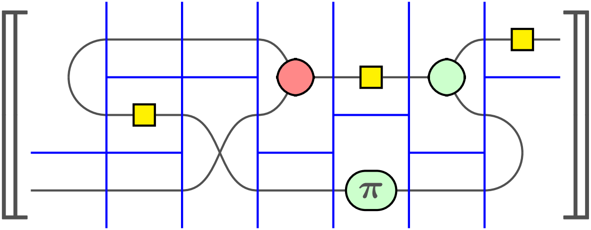

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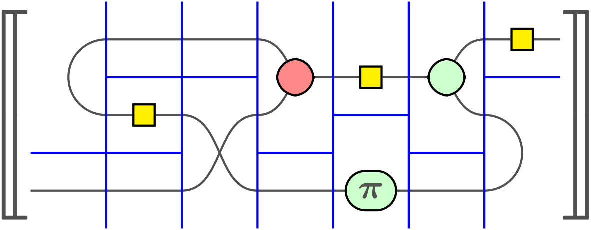

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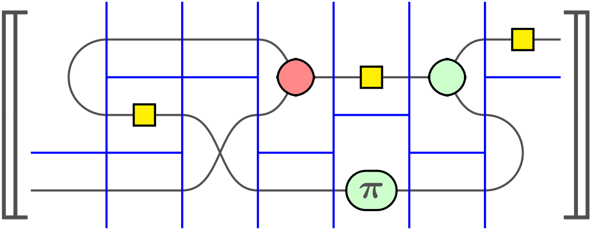

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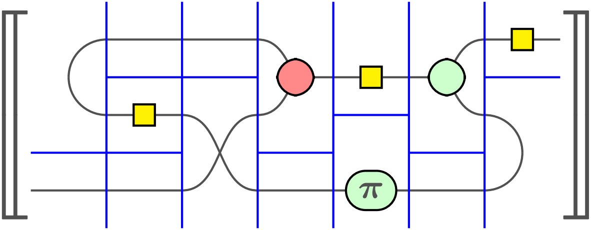

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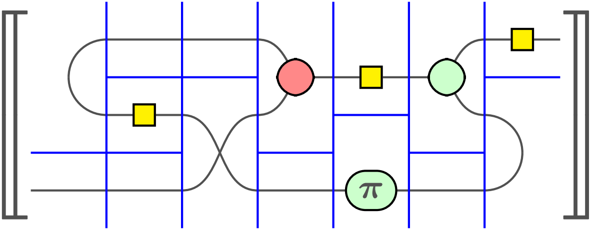

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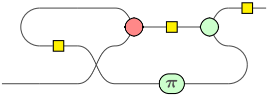
Rewriting rules



$\rightarrow$

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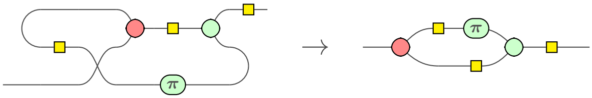

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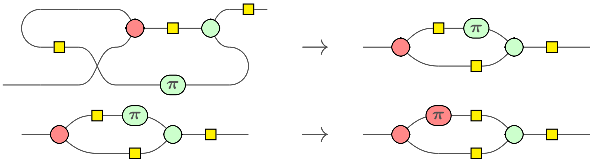

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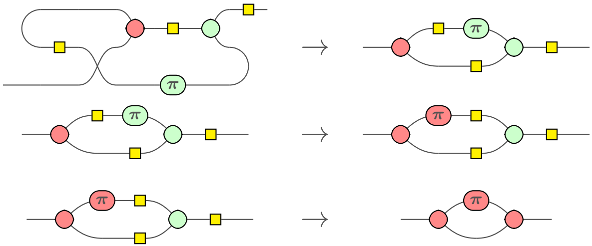

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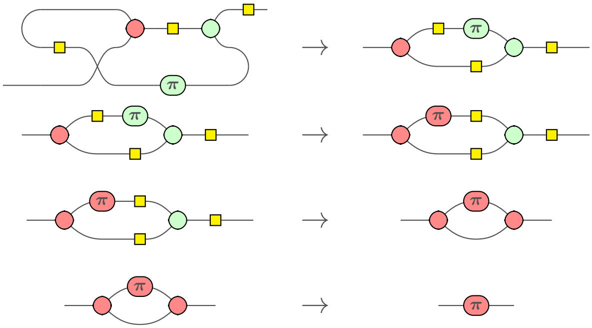

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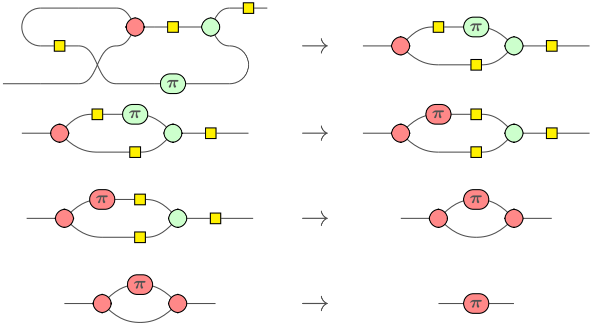
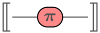
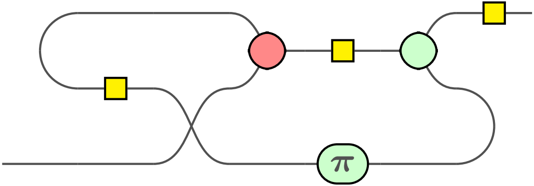




$$= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Check point



The ZX-calculus provides:

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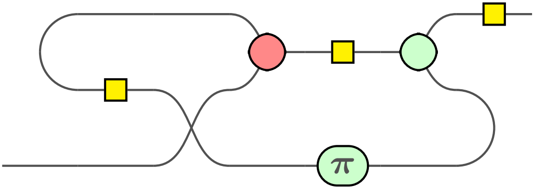

The ZX-calculus provides:

- Intuitive graphical calculus.

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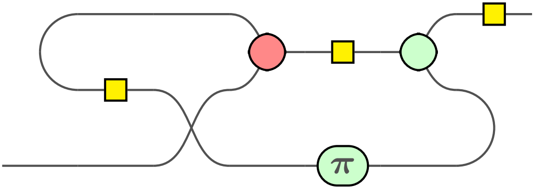

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- Universal and complete equational theory for **any number** of qubits. [FW18] and [JPV18].

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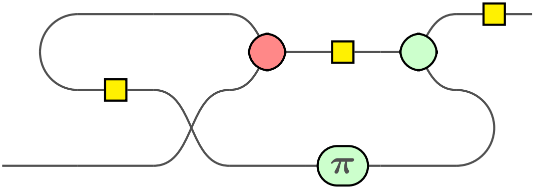

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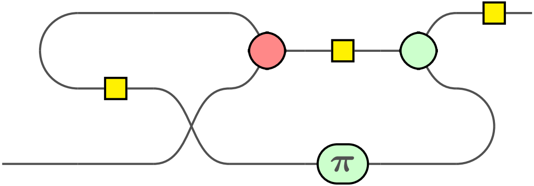

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We can be even more compact while scaling up the number of qubits!

Yes... but why?

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SZX-calculus I, Divide and Gather

$$2 \neq 1 + 1$$

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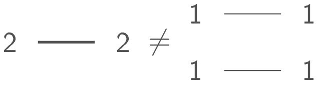

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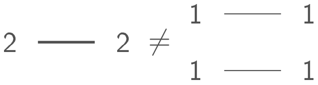

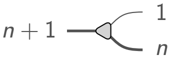

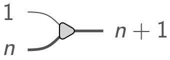

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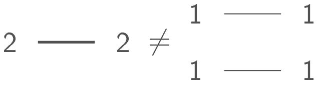

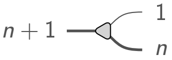

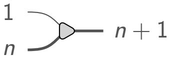

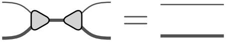

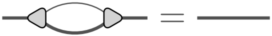

Theorem:

Let $\omega \in \mathbb{W}[a,b]$ and $\omega' \in \mathbb{W}[c,d]$:

$$\mathbb{W} \vdash \omega = \omega' \quad \Leftrightarrow \quad a = c \text{ and } b = d$$

SZX-calculus II. The rewiring theorem

Theorem:

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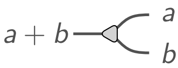

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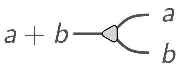

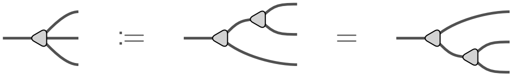

SZX-calculus, The big generators

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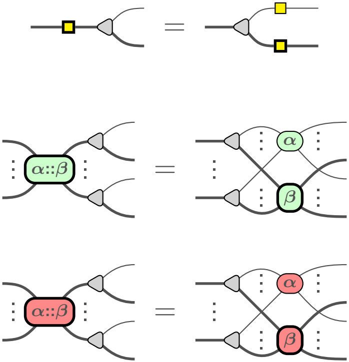

SZX-calculus, Completeness

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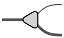
$[\![$  $]\!]_s \;:=\; (id_{n+1}, n+1, 1 \otimes n)$

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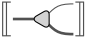
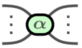
$[\![$  $]\!]_s := (id_{n+1}, n+1, 1 \otimes n)$

$[\![$  $]\!]_s := (\sum_{x\in\{0,1\}^n} e^{ix\bullet\alpha} |x^k\rangle\langle x^\ell|, n^{\otimes k}, n^{\otimes \ell})$

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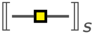

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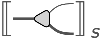

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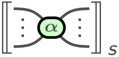

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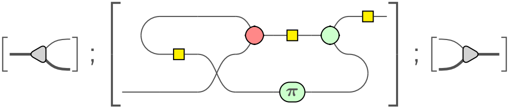
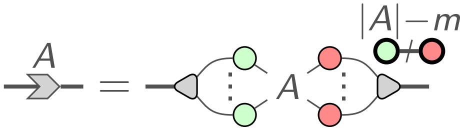


New large scale structures, Bipartite graph matrices.



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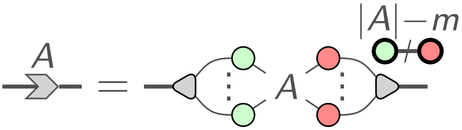

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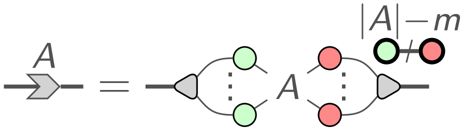

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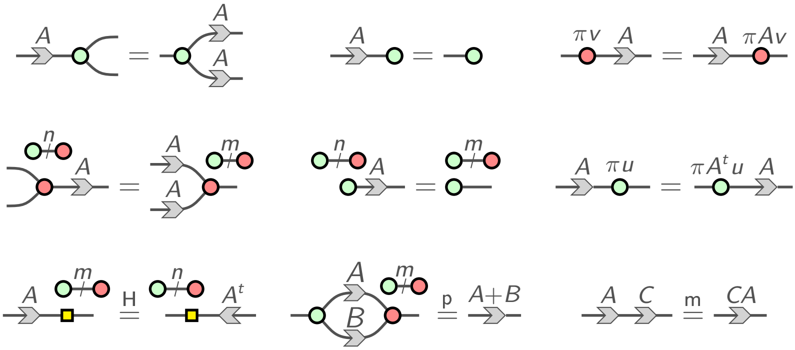
Properties of matrices



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The end.