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Contextual geometries

$$\begin{array}{c} X \otimes I - I \otimes X - X \otimes X \\ | \qquad | \qquad || \\ I \otimes Y - Y \otimes I - Y \otimes Y \\ | \qquad | \qquad || \\ X \otimes Y - Y \otimes X - Z \otimes Z \end{array}$$

Finite geometries $G = (\mathcal{O}, L)$ parametrized by observables $O \in \mathcal{O}$ with $\forall l \in L, l \neq \emptyset, l \subset \mathcal{O}$ such that $\forall l \in L, (\forall \{O_1, O_2\} \subset l, O_1 O_2 = O_2 O_1) \wedge (\prod_{O \in l} O = \pm I)$.

They are contextual if $\nexists f : \mathcal{O} \rightarrow \{-1, 1\} / \forall l \in L, \prod_{O \in l} f(O)I = \prod_{O \in l} O$. (1)

A linear problem

$$(\{-1, 1\}, \times) \rightarrow (\mathbb{Z}_2, +)$$

- Let A be the incidence matrix of G .
- Let e be the evaluation vector with $|L|$ entries such that, for the entry e_l corresponding to $l \in L, e_l I = \prod_{O \in l} O$

$$\text{Eq.(1) can be rewritten as } \nexists x \in (\mathbb{Z}_2)^n / Ax = e$$

Here x is in relation with f as follows: the entry corresponding to $l \in L$ would be $x_l = \prod_{O \in l} f(O)$.

Symplectic space

A space encoding the Pauli group

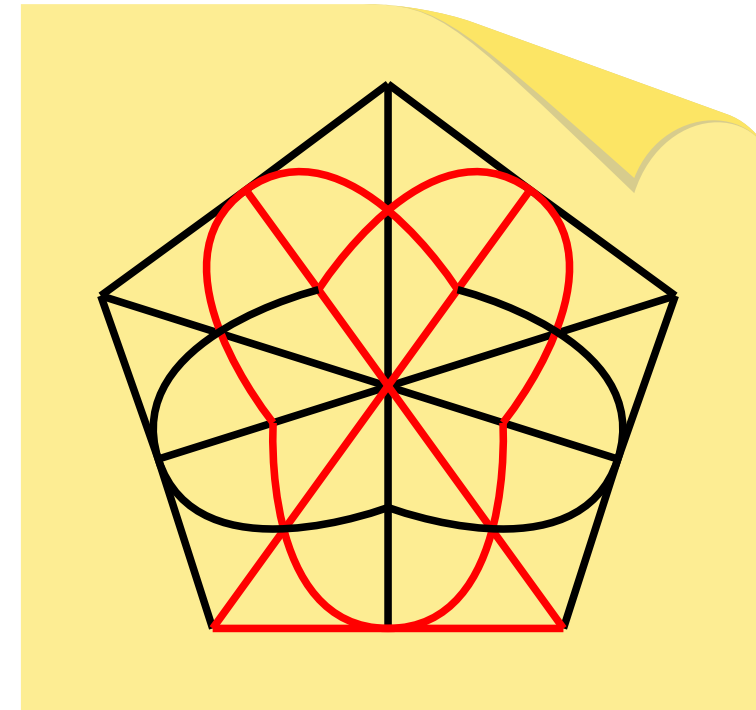
$$(0, 1, 1, 1) \leftrightarrow X \otimes Y$$

$$((0, 0) \leftrightarrow I, (0, 1) \leftrightarrow X, (1, 0) \leftrightarrow Z, (1, 1) \leftrightarrow Y)$$

With an inner product encoding commutativity

$$\left. \begin{array}{l} p_1 \leftrightarrow O_1 \\ p_2 \leftrightarrow O_2 \end{array} \right\} \Rightarrow (\langle p_1 | p_2 \rangle = 0 \Leftrightarrow O_1 O_2 = O_2 O_1)$$

We chose the lines of the geometry G to be subspaces of the symplectic space.



Results

A Magma program [dB21] generates all elements of five families of geometries and automatically establishes their contextuality:

Geometries	$n = 2$	$n = 3$	$n = 4$	$n = 5$
Lines	C(1)	C(1)	C(1)	C(1)
Generators	C(1)	N(1)	N(1)	N(1)
Hyperbolics	C(10)	C(36)	C(136)	C(528)
Elliptics	N/A (6)	C(28)	C(120)	C(496)
Perpsets	N(15)	N(63)	N(255)	N(1023)

n : Number of qubits of the system

N: Non-contextual

C: Contextual

(k): There are k instances in this family

N/A: Not applicable

bold: New results

References

[dB21] H. de Boutray Magma-contextuality: Automated detection of contextuality proofs with intermediate numbers of observables. *QuantCert*, <https://quantcert.github.io/Magma-contextuality/>.

[dHGM21] H. de Boutray, F. Holweck, A. Giorgetti, and P.-A. Masson, Automated synthesis of contextuality proofs from subspaces of symplectic polar spaces. *arXiv:2105.13798 [math-ph, physics:quant-ph]*, May 2021.

[Hol21] F. Holweck. Testing Quantum Contextuality of Binary Symplectic Polar Spaces on a Noisy Intermediate Scale Quantum Computer. *arXiv:2101.03812 [math-ph, physics:quant-ph]*, January 2021.

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